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Specific heat and entropy of fractional quantum Hall states in the second Landau level

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Specific heat has had an important role in the study of superfluidity and superconductivity, and could provide important information about the fractional quantum Hall effect as well. However, traditional measurements of the specific heat of a two-dimensional electron gas are difficult due to the large background contribution of the phonon bath, even at very low temperatures. Here, we report measurements of the specific heat per electron in the second Landau level by measuring the thermalization time between the electrons and phonons. We observe the activated behavior of the specific heat of the $\frac{5}{2}$ and $\frac{7}{3}$ fractional quantum Hall states, and extract the entropy by integrating over temperature. Our results are in excellent agreement with previous measurements of the entropy via longitudinal thermopower. Extending the technique to lower temperatures could lead to the detection of the non-Abelian entropy predicted for bulk quasiparticles at $\frac{5}{2}$ filling.

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Introduction. An ultraclean two-dimensional electron gas (2DEG) exposed to high magnetic fields and low temperatures plays host to a rich array of phases, including the intrinsically topological fractional quantum Hall (FQH) states. The $\nu = \frac{5}{2}$ FQH state is of particular interest, since it is believed to obey non-Abelian statistics [1,2]. Unfortunately, fabrication of devices to study the FQH states, such as quantum point contacts and interferometers, often degrades the quality of the sample, rendering the $\frac{5}{2}$ FQH effect unobservable. In cases where studies have been performed, their interpretation is difficult due to our incomplete understanding of the detailed physics of the quantum Hall edge. In this Rapid Communication, we introduce a technique to probe the bulk of the $\frac{5}{2}$ FQHE, avoiding the edge entirely. In particular, we report measurements of the specific heat at $\nu = \frac{5}{2}$, which, unlike transport, is sensitive to the total density of states (DOS). Moreover, one of the signatures of a non-Abelian system is an excess ground state entropy $S_{\mathrm{NA}} = k_B N_{\mathrm{qp}} \ln d$, where N_{qp} is the number of quasiparticles and d is the quantum dimension (equal to $\sqrt{2}$ for the conjectured non-Abelian Pfaffian and anti-Pfaffian states at $\frac{5}{2}$) [3,4]. In principle, this entropy could be detectable in the specific heat in the low-temperature limit [3].

Measurement of the specific heat of a 2DEG within a heterostructure is difficult because it is dwarfed by the contribution of the substrate. Earlier studies [5–7] applied conventional measurement techniques to multiple quantum well structures to boost the relative contribution of the electronic signal. In a more recent *tour de force* study, Schulze-Wischeler *et al.* [8] applied a phonon absorption technique to determine the specific heat in the FQH regime, albeit in arbitrary units. Our experiment similarly uses the weak electron-phonon coupling at low temperature to thermally isolate the 2DEG (on sufficiently short time scales), however we make use of *in situ* Joule heating and extract an absolute value for the specific heat. Furthermore, we use a Corbino disk, in which no edges connect the inner and outer contacts and we can be certain that we are probing the bulk of the 2DEG [9]. The radial symmetry of the Corbino geometry also simplifies analysis, since we can neglect the Nernst, Ettingshausen, and thermal Hall effects, which can strongly affect the temperature distribution in Hall and van der Pauw samples [10].

Experimental overview. Our experimental protocol consists of three conceptual parts, which are performed in an interlaced fashion in order to minimize the effects of drift. The first is to measure the conductance of the sample G as a function of electron temperature T_e, using an excitation small enough that T_e remains close to the refrigerator temperature T_0. At certain filling factors and temperatures, we find that G is highly temperature sensitive and thus can act as a thermometer for the 2DEG. Next, we apply a series of effectively dc voltage biases to the sample in order to simultaneously heat the 2DEG and measure its conductance. Using the calibration of $G(T_e)$, we deduce the electron temperature as a function of applied dc power and phonon temperature. From this we extract K, the total thermal conductance between the electronic system and the environment. Finally, we apply a singled-sided square wave at several kHz with a large (typically millivolt scale) bias V_{high}, as shown in Fig. 1(b). The thermal time constant τ can then be extracted from an exponential fit to the conductance transient response as shown in Fig. 1(c). This may be done either for the turn-on portion, as presented in the main body of this Rapid Communication, or as the electrons cool down again (using a small, but nonzero, bias for V_{low} as discussed in the Supplemental Material [11]). The total heat of the system can then be found from the relation $C = K\tau$ [12].

Experimental details. All measurements presented in this Rapid Communication were performed in a GaAs/AlGaAs heterostructure with a quantum well width 30 nm, electron density $n_e \approx 3.06 \times 10^{11}$ cm^{-2}, and wafer mobility 2.5×10^7 cm^2/V s measured at 0.3 K. The Corbino device was defined by a central contact with an outer radius $r_1 = 0.25$ mm and a ring contact with an inner radius $r_2 = 1.0$ mm. Full fabrication and characterization details can be found in Ref. [13].

Values of K are extracted from square wave response data. First, $G(T_0, P)$ is determined from the average of $I_{\mathrm{meas}}/V_{\mathrm{high}}$

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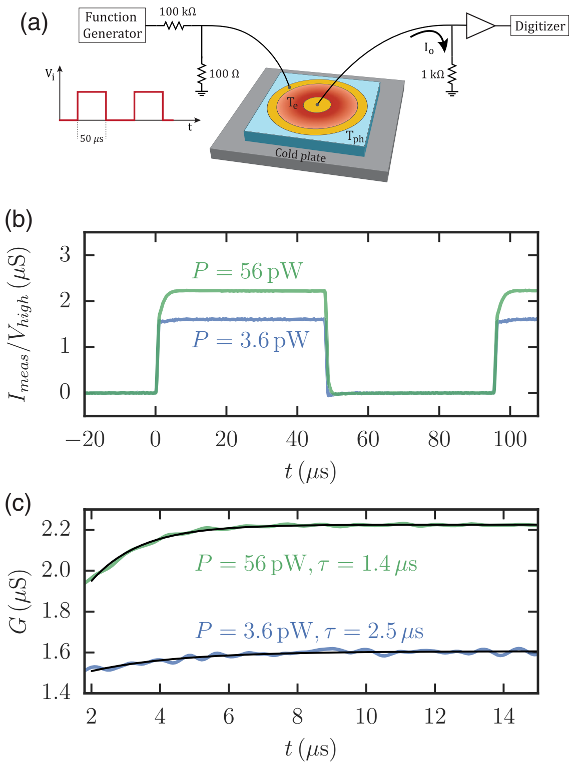

FIG. 1. (a) Experimental scheme. A square wave voltage bias is applied between the gold contacts (yellow) to heat the 2DEG (red) above the lattice temperature of the GaAs wafer (blue). The resulting current is measured across a 1 kΩ resistor to ground, using a voltage preamplifier and a digitizer. (b) Example time traces showing the measured current response. Each trace was obtained by averaging approximately 10^6 iterations. (c) Zoomed in plot of the conductance transient. Exponential fits are shown by the solid black lines, and corresponding values of τ are given for each curve.

after the thermal transient—for example, between $t = 15$ and 47 μs in Fig. 1(b). Then, a smooth cubic spline interpolation is fit to $T_{\rm ph}$ vs G in the low-power limit, where $T_e \simeq T_0$, and used to find $T_e(T_0, P)$ for higher heating powers. Finally, we calculate K from the slope of T_e vs P for P small enough to only raise the electron temperature by a few millikelvin. Further experimental details, including a correction factor for the Corbino geometry, are provided in the Supplemental Material [11].

The same data set used to find K is also used to find τ. As shown in Fig. 1(b), a transient is seen after the bias is turned on, but not when the voltage is turned off (since there is no voltage to convert the conductance into a current). Figure 1(c) shows an expanded view of the transient, after correction for possible *LRC* transients [11]. The fitted time constant is shorter at higher power, since the 2DEG reaches a higher temperature and therefore has a higher-energy emission rate. In order to find $\tau(T_e)$, we associate each measured time constant to the electron temperature inferred from the final conductance it reaches after several microseconds.

Results. We focus on several filling factors in the second Landau level (SLL) that are marked in Fig. 2(a), which shows the sample's conductance at the base temperature. Most of these filling factors are weakly gapped FQH states, however,

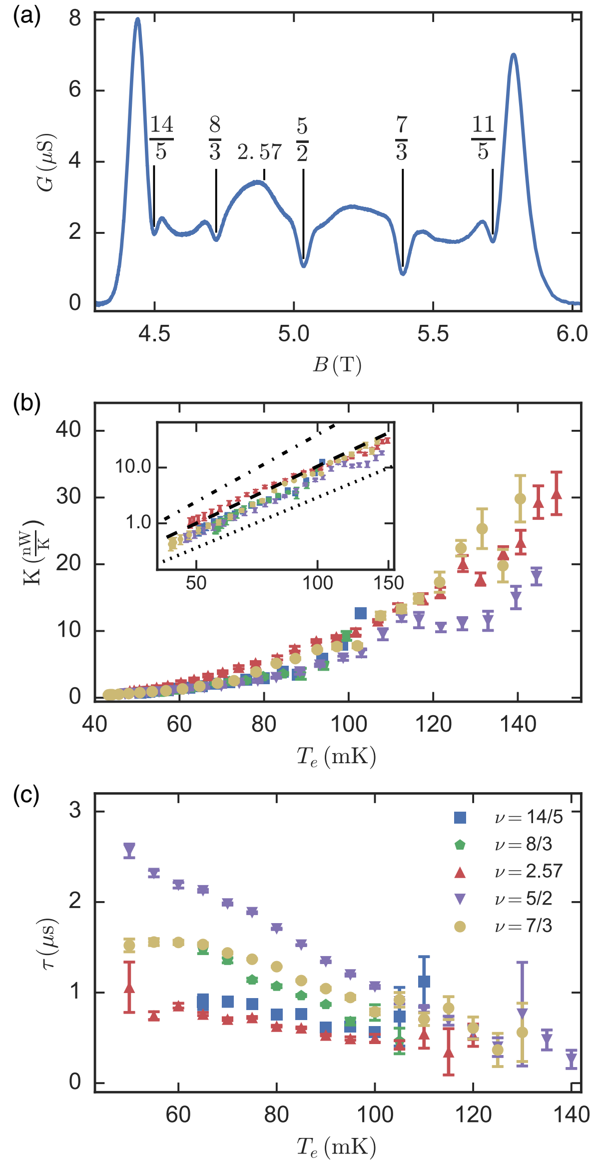

FIG. 2. (a) Conductance at base temperature. Arrows indicate filling factors where measurements of τ, K, and C were performed. (b) K vs T_e for several filling factors in the SLL. The legend is provided in (c). The inset shows the same data on a log-log scale with lines, at arbitrary vertical positions, indicating slopes of 3 (dotted), 3.4 (dashed), and 4 (dotted-dashed). (c) Thermal relaxation time τ measured as a function of T_e (as calculated from conductance measurements). Data from multiple phonon temperatures have been binned based on T_e in 5 mK bins and averaged.

we also measured at $\nu = 2.57$ where we observed a markedly increasing conductance with decreasing temperature. At lower temperatures, a reentrant integer quantum Hall state is often observed [14,15] at that same filling factor. Select results in high filling factors ($\nu > 10$) are presented in the Supplemental Material [11], with details to be presented in a separate publication.

Thermal conductance to the environment. Our results for K are shown in Fig. 2(b). The trend is similar for all of the filling factors shown, although $\nu = 2.57$ exhibits a somewhat higher

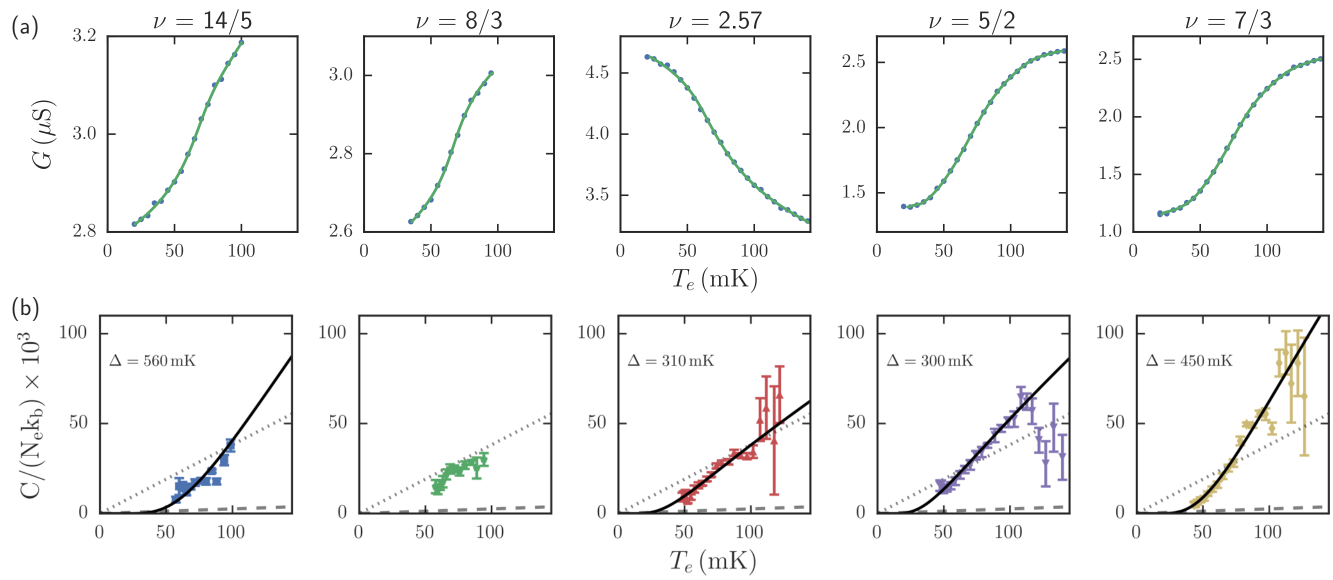

FIG. 3. (a) Conductance vs temperature for several states in the SLL. (b) Specific heat vs temperature at the same set of filling factors. Fits to Eq. (2) are shown by the solid black lines, and the resulting Δ's are given on each plot, while $g_0 k_B/n_e = 0.18$, 0.12, 0.16, and 0.25 K^{-1} for $\nu = \frac{14}{5}$, 2.57, $\frac{5}{2}$, and $\frac{7}{3}$, respectively. The dotted line is the specific heat for free 2D electrons ($m^* = m_e$), while the dashed line is the specific heat for 2D electrons in GaAs at zero field ($m^* = 0.067m_e$). The fit at $\nu = \frac{14}{5}$ is of lower quality than the others, and is discussed further in Ref. [11].

value of K throughout the temperature range below 100 mK. The magnitude of K is several orders of magnitude larger than would be expected according to the Wiedemann-Franz law ($K = 12GL_0T \approx 10$ fW/K, where L_0 is the Lorenz number and the factor of 12 arises from geometric considerations), effectively ruling out diffusion of charged quasiparticles to the contacts as a dominant cooling mechanism. Within the temperature range of our experiment, our results are instead consistent with cooling by phonon emission, although in principle we cannot rule out thermal transport by neutral quasiparticles [16].

At $B = 0$, the problem of electron-phonon power emission has been studied theoretically by Price and others [17,18], and good experimental agreement was found by Appleyard *et al.* [19]. Using that model, we would expect $K = 370T^4$ (nW/K) for our sample geometry, which is roughly two orders of magnitude lower than what we observe. However, this is consistent with the enhancement of composite fermion (CF)-phonon scattering (relative to electron-phonon scattering at $B = 0$) seen in experiments measuring the power emission power [20], phonon-drag thermopower [21], and phonon-limited mobility [22], as well as a theoretical treatment of CF-phonon interactions [23]. Fitting $K \propto T^n$, we find $n \simeq 3.4$ [indicated by the dashed black line in Fig. 2(c)], which is between the value $n = 4$ in the model by Price [17] and $n = 3$ in the hydrodynamic model put forward by Chow *et al.* [20]. Both of those models used a flat (metallic) DOS for the 2D electrons, and would have to be modified to take into account the gapped DOS at FQH states.

Thermal relaxation time. Figure 2(b) shows τ as a function of electron temperature for each state. These are direct measurements of electron-phonon energy relaxation times in the quantum Hall regime below 100 mK. Previous estimates of a T^4 or T^3 dependence for τ were based on measurements of dc electron-phonon energy emission rates and assumed linear behavior of $C(T)$ as calculated for a 2DEG at zero field [20,24,25]. Since we are measuring at (or near) gapped states, we do not expect $C(T)$ to be linear and we do not attempt to fit $\tau(T)$ with a simple power law. While τ is monotonically decreasing in all cases, there are clear differences between filling factors. The thermal relaxation time at $\nu = \frac{5}{2}$ is slower than those at $\nu = \frac{7}{3}$ and $\nu = \frac{8}{3}$, which are in turn slower than at $\nu = 2.57$ and $\nu = \frac{14}{5}$. The apparent differences in τ may be due to differences in the charge, size, and screening of quasiparticles at each filling factor.

Specific heat. The electronic heat capacity is now given by $C = K\tau$. Figure 3(c) shows the calculated specific heat $c \equiv C/k_B N_e$, where N_e is the total number of electrons in the Corbino disk. For comparison, we also show conductance versus temperature plots at each filling factor in Fig. 3(a). We begin our analysis by considering the specific heat of a Fermi liquid, given by

$$c = \frac{\pi m^* k_B T}{3\hbar^2 n_q}, \tag{1}$$

where m^* is the effective mass of the fermions and n_q is the number of quasiparticles. Using the band mass of GaAs, $m^* = 0.067m_e$, and $n_q = n_e$, we obtain the specific heat at $B = 0$, as shown by the dashed lines in Fig. 3(b). The observed specific heat is much larger—in fact, it agrees in magnitude with c for a Fermi liquid of free electrons [$m^* = m_e$, the dotted line in each panel of Fig. 3(b)]. This is in order-of-magnitude agreement with both theory [26] and experiments [27,28] that have shown composite fermions at half filling to have an effective mass close to that of free electrons. However, the specific heat at each filling factor increases rapidly in the region from 50 to 100 mK, exhibiting a strong deviation from linear (Fermi-liquid-like) behavior, as expected for a gapped DOS. For simplicity, we

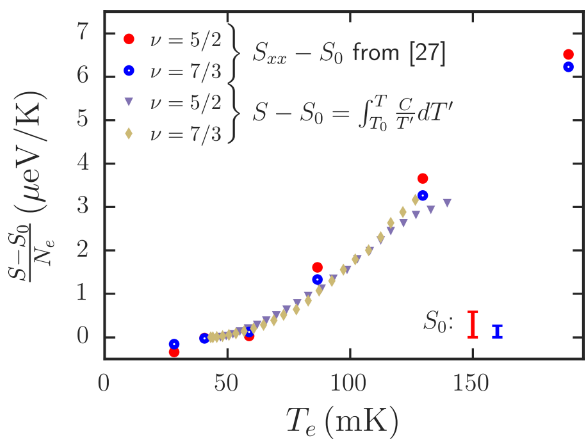

FIG. 4. Entropy as determined from longitudinal thermopower data [27] and from integration of our specific heat measurements. The thermopower data are offset such that $S - S_0 = 0$ at the lowest temperature for which we measured C. The offsets are $S_0 = 0.58$ and 0.26 for $\nu = \frac{5}{2}$ and $\frac{7}{3}$, respectively, as shown by the scale bars on the figure.

may consider the specific heat for a system of fermions with a toy DOS $g(\epsilon)$ given by two flat regions separated by a gap Δ, with the Fermi level exactly halfway between the two levels. From this model we obtain, in the low-temperature limit,

$$c = \frac{2k_B g_0}{n_e}\left(\frac{\Delta^2}{4k_B T} + \Delta + 2k_B T\right)e^{-\Delta/2k_B T}, \tag{2}$$

which is the equation used to fit the black curves in Fig. 3(b). In principle, the effective mass of the quasiparticles is given by $m^* = \pi\hbar^2 g_0$, and is found be $1.4m_e$ at $\frac{5}{2}$ and $2.1m_e$ at $\frac{7}{3}$. However, such analysis should only be taken as a rough estimate, since it is based on a simplified model of the DOS. Given the limited temperature range of our data set, a generic Arrhenius behavior also fits well to the data and yields a similar value for the gap energy [11]. Interestingly, standard Arrhenius fits to the conductance yield significantly smaller gap energies, specifically, $\Delta_{5/2} = 103$ mK and $\Delta_{7/3} = 131$ mK [11]. The discrepancy can be understood by considering more detailed models of conductance, such as the saddle-point model proposed by d'Ambrumenil *et al.* [29]. In their framework, the naive Arrhenius fit to conductance systematically underestimates the true energy gap. Using their recipe to estimate the true gap from conductance, we obtain $\Delta_{5/2} = 300$ mK and $\Delta_{7/3} = 330$ mK, in good agreement with the results from specific heat.

Entropy. The entropy of a 2DEG can also be accessed by measuring the longitudinal thermopower S_{xx}, which is related to entropy by the relation $S_{xx} = -S/(|e|n_e)$ in the clean limit [4]. In order to compare our results to existing thermopower data, we extract the entropy as a function of temperature using the formula

$$S(T) - S_0 = \int_{T_0}^{T} \frac{C}{T'} dT', \tag{3}$$

with T_0 being the lowest temperature at which we measured C. In Fig. 4, we plot $S(T) - S_0$, calculated from our measurements of C by numerical integration along with thermopower data obtained by Chickering *et al.* (Fig. 2 of Ref. [27]). The data from these two completely different techniques are in excellent agreement, suggesting that both do indeed measure the entropy of the electron system in the SLL.

The apparent activationlike behavior at $\nu = 2.57$ can also be understood by looking at the longitudinal thermopower data. Chickering *et al.* observed a step in S_{xx}, corresponding to the onset of the reentrant state and superlinearly increasing S_{xx} at higher temperature [27]. We do not observe the onset of the reentrant state itself, however, we do see superlinear (activationlike) behavior of c, in qualitative agreement with the thermopower result.

Conclusion. We have directly measured the electron-phonon energy relaxation rate and phonon emission power for several filling factors in the SLL. We observe a clear variation in the thermal relaxation times between filling factors, with $\nu = \frac{5}{2}$ in particular cooling more slowly than the other fractions in the SLL. We extract the specific heat for each filling factor, and find the expected activation behavior. Our results quantitatively agree with the entropy inferred from thermopower data, consistent with both techniques independently measuring the entropy of the 2DEG in the SLL. Further measurements at lower temperatures could be used to search for the non-Abelian entropy, and perhaps identify the degeneracy temperature for non-Abelian anyons at $\frac{5}{2}$.

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