Morphing of two-dimensional hole systems at $\nu = 3/2$ in parallel magnetic fields: Compressible, stripe, and fractional quantum Hall phases

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A transport study of two-dimensional (2D) holes confined to wide GaAs quantum wells provides a glimpse of a subtle competition between different many-body phases at a Landau level filling $\nu = 3/2$ in tilted magnetic fields. At large tilt angles (θ), an anisotropic, stripe (or nematic) phase replaces the isotropic compressible Fermi sea at $\nu = 3/2$ if the quantum well has a symmetric charge distribution. When the charge distribution is made asymmetric, instead of the stripe phase, an even-denominator fractional quantum state appears at $\nu = 3/2$ in a range of large θ, and reverts back to a compressible state at even higher θ. We attribute this remarkable evolution to the significant mixing of the excited and ground-state Landau levels of 2D hole systems in tilted fields.

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I. INTRODUCTION

A strong magnetic field perpendicular to a two-dimensional electron system (2DES) quantizes the electron kinetic energy into a set of highly degenerate Landau levels (LLs). The dominating Coulomb interaction then gives rise to numerous, exotic quantum many-body phases [1,2]. When the Fermi energy (E_F) lies in an $N = 0$ LL, there is a compressible Fermi sea of composite fermions at LL filling factors $\nu = \frac{1}{2}$ and 3/2 while numerous fractional quantum Hall states (FQHSs) are observed at nearby odd-denominator ν [1–5]. In $N \geqslant 2$ LLs, FQHSs are typically absent and anisotropic phases dominate at half-filled LLs, e.g., at $\nu = \frac{9}{2}$ and $\frac{11}{2}$ as the system breaks the rotational symmetry and forms unidirectional charge density waves—the so-called stripe (or nematic) phases [6–11]. The intermediate $N = 1$ LL is special. The electrons exhibit FQHSs not only at odd-denominator ν but also at the even-denominator fillings $\nu = \frac{5}{2}$ and $\frac{7}{2}$ [1,2,12]. The latter are believed to be the Moore-Read Pfaffian state [13], obey non-Abelian statistics, and be of potential use in topological quantum computing [14]. The application of a symmetry-breaking parallel magnetic field ($B_\parallel$), or isotropic hydrostatic pressure which introduces LL mixing, can lead to the destruction of the $\nu = \frac{5}{2}$ FQHS and stabilization of the stripe phase in the $N = 1$ LL [15–19].

In GaAs two-dimensional hole systems (2DHSs), the spin-orbit coupling mixes harmonic Landau oscillators with different orbital and spin indices and leads to a complex set of LLs [20,21]. Nevertheless, in narrow quantum wells (QWs), the 2DHS is compressible at $\nu = \frac{1}{2}$ and 3/2 and numerous odd-denominator FQHSs are still prevalent as the filling deviates from $\nu = \frac{1}{2}$ and 3/2, qualitatively similar to those in 2DESs. However, the even-denominator FQHSs at $\nu = \frac{5}{2}$ and $\frac{7}{2}$ are very weak [22,23], and instead stripe phases are typically observed at these fillings, particularly at low densities [23–25]. Here, we report transport measurements in 2DHSs confined in *wide* GaAs QWs and subjected to strong $B_\parallel$. We observe a remarkable metamorphosis of the ground state at $\nu = 3/2$. The compressible Fermi sea seen at $\nu = 3/2$ turns into a stripe phase when we apply a sufficiently large $B_\parallel$ to a symmetric QW. The stripe phase can be destabilized in asymmetric QWs and, strikingly, an even-denominator FQHS forms at $\nu = 3/2$ at intermediate $B_\parallel$. At larger $B_\parallel$, the $\nu = 3/2$ FQHS disappears and the 2DHS reverts back to becoming compressible. Our results highlight the rich and subtle many-body phenomena manifested by high-quality 2DHSs.

II. METHOD

Our samples were grown by molecular beam epitaxy, and each consists of a GaAs QW (well widths $W = 35$ or 30 nm) which is bounded on either side by undoped $Al_{0.3}Ga_{0.7}As$ spacer layers and C δ-doped layers. They have as grown densities $p \simeq$ 1–1.5 $\times 10^{11}$ cm^{-2} and high mobility $\mu \simeq$ 100 m^2/V s. Each sample has a van der Pauw geometry, with alloyed InZn contacts at the four corners of a 4×4 mm^2 piece. We carefully control the density and the charge distribution symmetry in the QW by applying voltage biases to the back and front gates [26–28]. For the low-temperature measurements, we use a dilution refrigerator with a sample platform which can be rotated *in situ* in the magnetic field to induce a parallel field component $B_\parallel$ along the x direction [see Fig. 1(c)]. We use θ to express the angle between the field and the normal to the sample plane, and denote the longitudinal resistances measured along and perpendicular to the direction of $B_\parallel$ as R_{xx} and R_{yy}, respectively [Fig. 1(c)].

III. DATA AND DISCUSSION

Although the main focus of our study is the state of the 2DHS near $\nu = 3/2$ in *tilted* magnetic fields, the data at $\theta = 0$ are also very intriguing. Figure 2 shows R_{xx} measured from a symmetric 35-nm-QW 2DHS at $\theta = 0^\circ$ and different densities. Strong odd-denominator FQHSs are seen as vertical, low-resistance (blue) stripes at $\nu = \frac{5}{3}, \frac{8}{5}, \frac{7}{5}$, and $\frac{4}{3}$. With increasing density, R_{xx} steeply increases above a boundary marked by the white solid line. This sharp transition is a signature of a LL crossing near $\nu = 3/2$. We indeed expect such a crossing from the typical LL diagram [see Fig. 1(a)] for our wide-QW 2DHSs [29]. As depicted in Fig. 1(a), the light-hole-like β level (blue) crosses the heavy-hole-like α level (green) as a function of $B_\perp$, and the Fermi energy E_F at $\nu = 3/2$ moves from the α to β level as we increase the density. Because of the strong mixing of the heavy-hole and light-hole Landau oscillators through the spin-orbit interaction, the LLs for holes confined to wide GaAs QWs are nonlinear with $B_\perp$ and show multiple crossings [20,21]. In previous studies, the

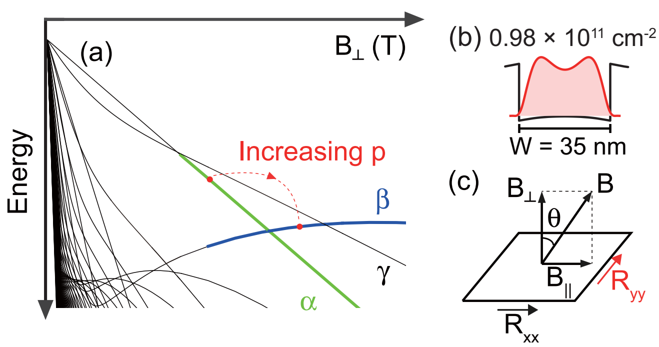

FIG. 1. (a) Schematic LL diagram. The light-hole-like β level (blue) crosses the heavy-hole-like α level (green) as a function of $B_\perp$. As indicated by the red arrow, the Fermi energy E_F at $\nu = 3/2$ moves from the α to the β level as we increase the density. (b) Calculated charge distribution and potential of the 35-nm-QW 2DHS at $p = 0.98 \times 10^{11}$ cm^{-2}. (c) Measurement setup. $B_\parallel$ is applied by tilting the sample in field, and R_{xx} and R_{yy} denote the longitudinal resistance measured along and perpendicular to $B_\parallel$.

crossing between the *lowest* two LLs, those labeled β and γ in Fig. 1(a), was shown to lead to novel many-body phenomena, such as the stabilization of two-component QHSs at $\nu = \frac{1}{2}$ and $\nu = 1$, as well as the appearance of an anisotropic Wigner crystal near $\nu = \frac{1}{2}$ [30–33]. Here we focus on data taken near the LL crossing at $\nu = 3/2$.

In Fig. 1(a) we denote the two crossing LLs near $\nu = 3/2$ by α and β. According to the calculations, the α level is a heavy-hole LL, and has a pure orbital ($N = 0$) and spin ($J_z = -3/2$) character. It does not mix with other LLs at $\theta = 0$. The β level, on the other hand, has a light-hole character, and has mixing from the higher-index LLs. At low densities and small magnetic fields, i.e., below the boundary marked by the white line in Fig. 2, E_F at and near $\nu = 3/2$ is in the α level. When p is increased, E_F moves to the β level above the boundary. The FQHSs at $\nu = \frac{5}{3}$ and $\frac{4}{3}$ are seen on both sides of the crossing. They do not disappear at the crossing, although are clearly weaker above the white boundary. This is qualitatively similar to what is seen at the crossing of two parallel-spin LLs, and is in contrast to the crossing of two opposite-spin LLs where the FQHSs become very weak or disappear [34–37]. We conclude that the two crossing LLs at $\nu = 3/2$ in Fig. 2 have similar spin (or pseudospin) orientations. Also our observation of well-developed $\nu = \frac{7}{5}$ and $\frac{8}{5}$ FQHSs on both sides of the white boundary (see R_{xx} and R_{yy} traces at $p \simeq 0.98$ and 1.35×10^{11} cm^{-2} included in Fig. 2) implies that the two crossing levels have a primarily $N = 0$ character. Note that the FQHSs at similar fillings in an $N = 1$ LL (e.g., the $\nu = \frac{12}{5}$ FQHS in GaAs 2D electrons) are extremely fragile and are only seen in the highest-quality samples and at extremely low temperatures [38,39]. While the $\theta = 0$ data of Fig. 2 strongly suggest that the two crossing levels near $\nu = 3/2$ have an $N = 0$ character, we emphasize that at high $B_\parallel$ where we observe the new phases of the 2DHS at $\nu = 3/2$, the 2DHS LLs are very likely to be mixed with higher-index LLs.

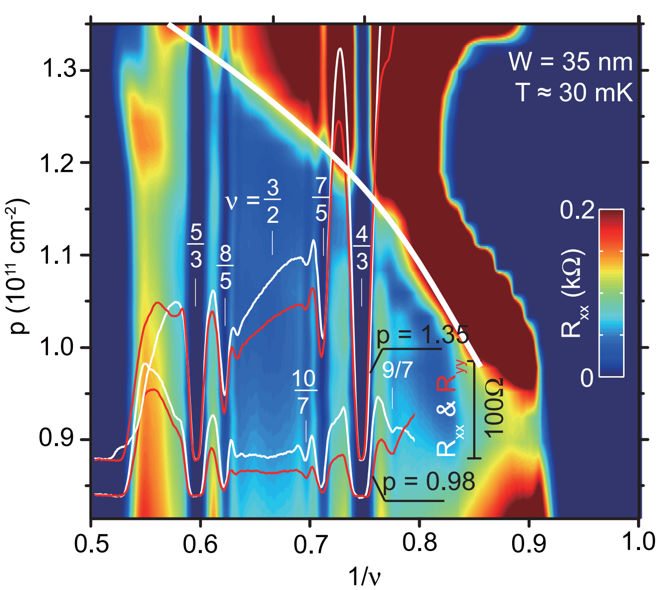

FIG. 2. Color-coded plot of R_{xx} measured from the 35-nm-wide QW sample at $\theta = 0^\circ$ and different densities. A sharp phase transition can be seen as a steep increase of R_{xx} at the boundary marked by the solid white line, signaling a LL crossing. The R_{xx} (white) and R_{yy} (red) traces are taken at $p = 0.98$ and 1.35×10^{11} cm^{-2}, before and after the crossing, respectively. Despite the significant increase in R_{xx} and R_{yy} at $p = 1.35 \times 10^{11}$ cm^{-2}, the 2DHS in both cases is isotropic and exhibits strong FQHSs at $\nu = \frac{4}{3}, \frac{5}{3}$, etc., and a compressible Fermi sea at $\nu = 3/2$, indicating the two crossing LLs are both $N = 0$.

We now concentrate on data taken in tilted fields. Figure 3(a) shows data from the 35-nm-wide QW at $p \simeq 0.98 \times 10^{11}$ cm^{-2} and for different tilt angles θ [40]. The traces presented in this figure were taken at $T \simeq 50$ mK, somewhat higher than in Fig. 2 data, because of the higher base temperature of the dilution refrigerator with the rotatable platform. The large-θ traces in Fig. 3(a) highlight our first key observation: the $B_\parallel$-induced transition from a compressible to a stripe phase at $\nu = 3/2$ [41]. The $\nu = 3/2$ compressible state and the surrounding odd-denominator FQHSs remain stable as we increase θ to $\sim 56^\circ$. Data at $\theta \simeq 65^\circ$, however, reveal a weakening of the FQHSs and a very strong transport anisotropy: R_{xx} at $\nu = 3/2$ shows a peak that is about two orders of magnitude larger than R_{yy}, which exhibits a minimum. These features have a striking resemblance to what is seen when stripe phases form in GaAs 2DESs at large $B_\parallel$ in $N \geqslant 1$ LLs ($\nu \geqslant \frac{5}{2}$) [15,16]. Note that the "hard axis" in Fig. 3(a) data is along the $B_\parallel$, also similar to the 2DES data in previous experiments [15,16]. The surprise is that in our 2DHSs samples we observe stripe-phase features at $\nu = 3/2$, and when the relevant LLs clearly have an $N = 0$ character at $\theta = 0^\circ$. We conclude that $B_\parallel$ induces a severe mixing of the higher-index LLs, leading to the stripe phase at fillings as small as $\nu = 3/2$.

In Fig. 3(b), we show data for a 30-nm-QW sample with $p \simeq 1.56 \times 10^{11}$ cm^{-2} at $\theta = 0^\circ$ and 65°; data at $\theta = 66^\circ$ are taken from another sample with a slightly higher density $p \simeq 1.59 \times 10^{11}$ cm^{-2}. The charge distributions in these samples are symmetric and the 2DHSs in both samples show a strong FQHS at $\nu = \frac{1}{2}$ when $\theta = 0^\circ$. In both samples, the presence of an anisotropic phase at $\nu = 3/2$ at large θ is clear. The similarity of the $\theta = 66^\circ$ traces to those at $\theta = 65^\circ$ for the 35-nm-sample [Fig. 3(a)] is remarkable. In both cases, the system shows a switching of the anisotropy direction as a

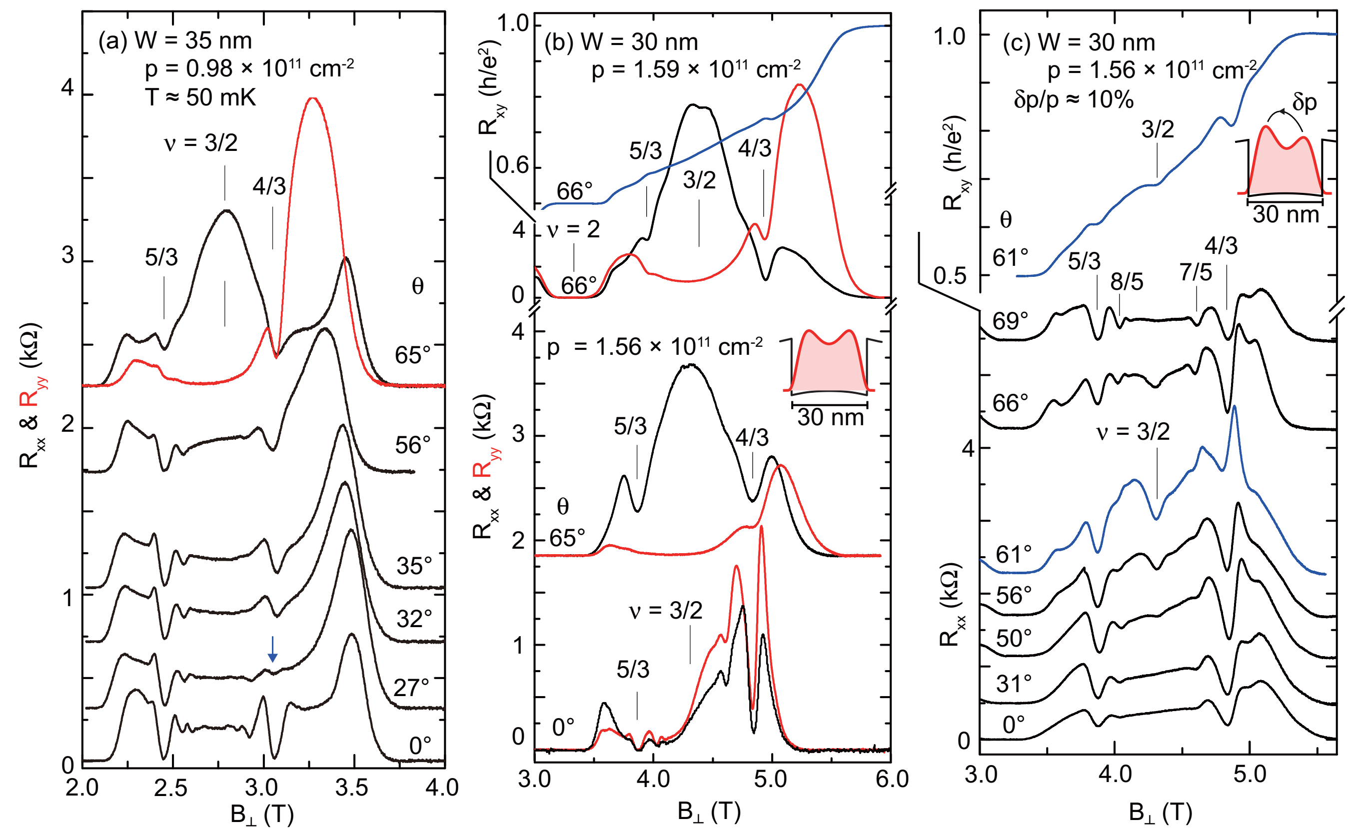

FIG. 3. (a) R_{xx} and R_{yy} for the 35-nm-QW 2DHS at density $p \simeq 0.98 \times 10^{11}\,\text{cm}^{-2}$ and different tilt angles (θ). At $\theta = 65^\circ$, the compressible Fermi sea at $\nu = 3/2$ turns into a stripe phase and the R_{xx} minima at $\nu = \frac{4}{3}$ and $\frac{5}{3}$ become weak. (b) Magnetoresistances for the symmetric 30-nm-QW samples. (c) In a slightly asymmetric QW, no stripe phase but a Fermi sea is seen at very large θ. Remarkably, near $\theta \simeq 61^\circ$, an even-denominator FQHS is seen at $\nu = 3/2$. In all panels, the traces are shifted vertically for clarity.

function of filling factor: $R_{xx} \gg R_{yy}$ near $\nu = 3/2$ but $R_{xx} \ll R_{yy}$ for $1 \lesssim \nu \lesssim \frac{4}{3}$ ($6 \gtrsim B_\perp \gtrsim 5$ T). The $\theta = 65^\circ$ traces in Fig. 3(b) also clearly show an anisotropic phase at $\nu = 3/2$, but for $B_\perp \gtrsim 5$ T ($\nu \lesssim \frac{4}{3}$), transport is essentially isotropic. We do not know the origin of the "switching" behavior or the difference between the Fig. 3(b) samples [42]. Regardless of the difference, it is clear that in all three samples of Figs. 3(a) and 3(b), $R_{xx} \gg R_{yy}$ at $\nu = 3/2$ at large θ. The data provide strong evidence for a transition from an essentially isotropic, compressible phase at $\nu = 3/2$ at $\theta = 0$ to an anisotropic phase at large tilt angles. Note also that in all these three samples, at $\theta \simeq 66^\circ$ the longitudinal resistances exhibit minima and the Hall resistance exhibits developing plateaus at $\nu = \frac{4}{3}$. The coexistence of a stripe phase and a FQHS at a nearby filling is noteworthy. Our data (not shown) also indicate that the anisotropic phase only appears at very low temperatures; when we raise T to $\simeq$200 mK, R_{xx} and R_{yy} become nearly isotropic. This is similar to what is observed for $B_\parallel$-induced anisotropic phases, e.g., at $\nu = \frac{5}{2}$, in GaAs 2D electrons [15,16,18,19].

Data presented in Fig. 3(c) reveal yet another twist in the fate of the ground state at $\nu = 3/2$ in 2DHSs. Here we show data for a third 30-nm-QW sample. It is from the same wafer as the Fig. 3(b) data and has a very similar density ($p \simeq 1.56 \times 10^{11}$ cm^{-2}) but the charge distribution is slightly asymmetric [about 10%; see the inset in Fig. 3(c)] as judged from the absence of a FQHS at $\nu = \frac{1}{2}$ at $\theta = 0^\circ$ [43]. Surprisingly, when tilted to high θ, there is no anisotropic phase; instead, an even-denominator FQHS develops at $\nu = 3/2$. The state is strongest at $\theta = 61^\circ$ with a deep R_{xx} minimum accompanied by a clear R_{xy} plateau at $\frac{2}{3}(h/e^2)$. At higher θ the $\nu = 3/2$ FQHS becomes weaker and completely disappears at $\theta = 69^\circ$ where it is replaced by a compressible state. The $\theta = 69^\circ$ trace is by itself intriguing as the FQHSs at $\nu = \frac{4}{3}$ and $\frac{5}{3}$ are quite strong, even stronger than they are at $\theta = 0^\circ$, and there are also developing higher-order FQHSs at $\nu = \frac{7}{5}$ and $\frac{8}{5}$.

The unusual evolution we observe for the ground state as a function of θ and charge distribution symmetry attests to the subtle and complex nature of 2DHS LLs in the presence of both perpendicular and parallel magnetic fields. Unfortunately, calculations of the LLs in multisubband 2DHSs at finite θ are very challenging and unavailable; indeed, we hope that our results would provide a stimulus for such calculations. We note that in GaAs 2DESs, no stripe phases are observed, or expected, when E_F lies in an $N = 0$ LL. Our data therefore suggest that there is a significant mixing of the higher-index LLs at large $B_\parallel$ in 2DHSs confined to wide QWs, and the mixing leads to a close competition between various many-body states, including the stripe phase and the even-denominator FQHS. Given the complex hole band structure and LLs, such mixing is certainly plausible.

The FQHS we observe at $\nu = 3/2$ is particularly intriguing. In very-high-quality, single-layer 2D systems, even-denominator FQHSs are seen at $\nu = \frac{5}{2}$ (and $\frac{7}{2}$) when E_F lies in an $N = 1$ LL [12,22,44]. The origin of these FQHSs is not yet fully established. They might be a one-component, Moore-Read Pfaffian state [13] and obey non-Abelian statistics [14], or be the two-component (Ψ_{331}) state [45]. In 2D systems with a layer or subband degree of freedom, even-denominator FQHSs are also seen at $\nu = \frac{1}{2}$ (and 3/2) when the $N = 0$ LLs from different layers or subbands are close in energy [26–28,32,46,47]. These are generally believed to be two-component FQHSs.

Compared to the even-denominator FQHSs described above, the $\nu = 3/2$ FQHS we observe near $\theta \simeq 61^\circ$ in Fig. 3(c) has some unique characteristics. For example, the two-component $\nu = \frac{1}{2}$ state seen in wide QWs is strongest when the charge distribution is symmetric, and turns into a compressible state for sufficient (∼10%) charge distribution asymmetry [26–28]. This is opposite to what we see in Figs. 3(b) and 3(c), where the charge distribution asymmetry *helps* to stabilize the 3/2 FQHS. As another example, we note that the $\frac{5}{2}$ FQHS in GaAs 2DESs is observed at $\theta = 0$ but becomes unstable at large $B_\parallel$ and turns into a stripe phase. Again, this is in contrast to the behavior in Fig. 3(c). We suggest that it is the mixing of $N = 1$ and $N = 0$ LLs that leads to the stabilization of the 3/2 FQHS at intermediate θ in Fig. 3(c). There is indeed experimental evidence [34], backed by theoretical calculations [48], that the $\nu = \frac{5}{2}$ FQHS, which is formed in an $N = 1$ LL, becomes more robust when there is a nearby $N = 0$ LL.

Finally, a FQHS at $\nu = 3/2$ was observed in ZnO samples and was interpreted to result from E_F lying in an $N = 1$ LL [44]. This state also appears in systems with asymmetric charge distribution, and only in a finite range of $\theta \simeq 40^\circ$. There are, however, some important differences between the $\nu = 3/2$ FQHSs in ZnO samples and in our 2DHSs. First, the ZnO $\nu = 3/2$ FQHS appears in the vicinity of a LL crossing, and the transitions of the nearby odd-denominator FQHSs are also consistent with the pseudospin polarization transitions documented in Ref. [37]. In contrast, Fig. 3(c) data exhibit no clear signature of a LL crossing or the related FQHS transitions. Second, the $\nu = 3/2$ FQHS in ZnO is observed in a narrow range of θ, and then the system turns insulating at higher angles [44]. In our 2DHS, on the other hand, once the $\nu = 3/2$ FQHS disappears at the largest θ, the system exhibits a compressible phase at $\nu = 3/2$, surrounded by strong, odd-denominator, FQHSs [see the upper R_{xx} trace in Fig. 3(c)].

IV. CONCLUSION

In conclusion, our results for 2DHSs confined in wide GaAs QWs reveal a remarkable evolution of different many-body phases at $\nu = 3/2$ in tilted magnetic fields: a compressible state, an anisotropic (stripe) phase, and an incompressible FQHS. We attribute this metamorphosis to the subtle nature of the mixed 2DHS LLs in tilted fields. The detailed character of these LLs and how they affect the ground states of the 2DHS await future theoretical and experimental studies. We also note that in our 2DHSs, the competition at the highest angles between the stripe phase and the compressible phase appears to be correlated with the absence or presence of the $\nu = 3/2$ FQHS at lower angles which in turn depend on the symmetry of the charge distribution.

Note added. Recently, we became aware of a related study, where anisotropic phases at large θ are also reported near $\nu = 3/2$ in 2DHSs confined in narrower GaAs QWs [49].

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[40] The $\nu = \frac{4}{3}$ FQHS nearly disappears at $\theta = 27^\circ$ and is then restored at $\theta \gtrsim 32^\circ$. A similar transition is seen as a function of density in a 30-nm-QW 2DHS at $\theta = 0$ and $p \simeq 1.20 \times 10^{11}\text{cm}^{-2}$. This is likely a FQHS spin-polarization transition. It is *unrelated* to the crossing of the α and β levels, because otherwise a transition would appear at $\nu = \frac{5}{3}$ instead [37]. Note that we observe no spin-polarization transition of the $\nu = \frac{4}{3}$ FQHS in Fig. 2(a) data taken from the 35-nm QW as a function of increasing p. This difference is likely a result of complex LL-mixing mechanisms in our 2DHSs. The FQHS spin-polarization transitions of 2DHSs deserve more systematic studies.
[41] We also observe stripe phases at $\nu = 3/2$ at large θ in symmetric 20- and 40-nm-wide GaAs QW samples.
[42] We emphasize that the difference between the $\theta = 65^\circ$ and $\theta = 66^\circ$ traces in Fig. 3(b) is intrinsic to the two samples and not because of the slightly different angles.
[43] In Fig. 3(b), at $\theta = 0^\circ$, R_{xx} at $\nu < 3/2$ becomes much larger than at $\nu > 3/2$ and the $\nu = \frac{4}{3}$ FQHS is much weaker than the $\nu = \frac{5}{3}$ FQHS. Based on Fig. 2 data, we conclude that a crossing between the α and β levels is occurring for the sample of Fig. 3(b) near $\nu = 3/2$. For the asymmetric QW [Fig. 3(c)], we do not see such a strong rise of resistance near $\nu = 3/2$. It is possible that the finite Rashba effect in the asymmetric QW couples the α and β levels through the mixing between the heavy and light holes, thus removing the crossing and the transition near $\nu = 3/2$.
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